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INTERVAL-VALUED INTUITIONISTIC FUZZY PATTERN RECOGNITION MODEL FOR ASSESSMENT OF SOCIAL COHESION

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Social cohesion is defined as the potential of a society to sustain the well-being, eliminate inequality, ensure the rights for every citizen, respect for dignity, the opportunities for human development and realization, and engagement of all individuals in the democratic system. There exist numerous researches in this direction differing in the method, structure and number of indicators constituting Social Cohesion Index (SCI). In the present study, we developed an approach based on interval-valued intuitionistic fuzzy tools for the assessment of SCI. In the adoption of the structure of SCI, we relied on the UN methodology. The advantages of the proposed approach are in taking into account the uncertainty caused by crisp input data and classical computation techniques. The issues addressed in the research encompass the effect of indicators on the overall SCI, computation of the weights of indicators and sub-indices, producing the aggregated index and assessing its level through fuzzy pattern recognition tools. The approach proposed in the current work can be a substantial advance in the methodology of SCI calculations.

Keywords: *social cohesion, interval-valued intuitionistic fuzzy number, pattern recognition*

JEL Classification: A13, C65, Z13

1. Introduction

The concept of social cohesion was firstly used by sociologist Durkheim. He viewed social cohesion as an ordering characteristic of a society and outlined it as the relations, shared loyalties and solidarity between society members [1]. Social cohesion is often described as the potence of social relationships, shared values, sense of belonging, trust, extent of inequality and disparity in society [1, 2].

The studies on the domain of social relations firstly involves its quality and quantity measurements [3-5]. Secondly, the literature regarding the cohesion addresses issues of belonging and identification in social groups and the equality perception in society [6-8]. Finally, the literature reveals that focus on the common good covers: a recognition and the constitution of a social order, standards and rules [1, 9, 10]; socio-cultural norms and participation in political life [11].

Moreover, Schiefer et al. [12] distinguished two additional concepts in the literature connected to social cohesion: (in)equality, and objective and subjective quality of life. The (in)equality concept is studied in the literature on three separate measures: first, the (un)equal allocation of resources within and across societies; second, equal opportunities particularly in terms of employment, income, health, education, rights, and social services; and finally, the exclusion of certain groups from social life [11]. However, objective and subjective quality of life encompasses general well-being as happiness and life satisfaction, health, and the objective living standards of certain social groups.

The Hong Kong Council of Social Service [13] proposed the following social cohesion indicators:

- bonding: family/friends/neighbors;
- bridging: community/society;
- linking: government/governed.

Council of Europe’s Strategy (CES) defines social cohesion as the potential of a society to ensure the welfare, eliminate disparities, ensure the rights for every citizen, respect for dignity, the opportunity for personal development, and participation in the democratic processes for all members of society. It also takes into account the policies on employment, education and health, as well as on vulnerable groups as immigrants and disabled people. CES also emphasizes the significance of values in political choices, and takes them into account when developing indicators [14].

Economic Commission for Latin America and the Caribbean (ECLAC) developed social cohesion indicators [15], presented in Table 1.

Table 1

SYSTEM OF SOCIAL COHESION INDICATORS: COMPONENTS AND FACTORS (ECLAC)

Indicators		
Gaps	**Institutions**	**Belonging**
Income inequality Poverty and indigence Employment Education Health Housing Pensions Digital divide	Effectiveness of democracy State institutions Market institutions Family	Multiculturalism Trust Participation Expectations of mobility Social solidarity

Burns et al. [16] defined social cohesion as the extent to which people are co-operative within and over group boundaries, apart from coercion or purely self-centered motivation.

Social cohesion is a broad concept that encompasses social capital too. It covers not only mutual relations, but also equal and righteous treatment, respect, and care. This intrinsically requires an equitable distribution of resources and opportunities between individuals that can strengthen bonds across demographic strata [17].

Social cohesion is often regarded as a crucial measure of inclusiveness and equal opportunities within a society. It encompasses various indicators, such as life satisfaction, income inequality, trust in government and institutions, civic participation, and social mobility, which collectively gauge the degree of social connectedness and the ability of a society to offer equitable opportunities to its members [18, 19].

Considering the importance of taking into account in the SCI calculation a wide range of quantitative and qualitative indicators, characterized by a high degree of uncertainty, in the work it was decided to use the tools of fuzzy sets, well adapted to working with this kind of information. As an extension of intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets (IVIFS) are more efficient to deal with uncertainty [20, 21]. In the empirical part of the study, an algorithm was developed for calculating the SCI, resorting to interval-valued intuitionistic fuzzy multiple-criteria decision-making methods (MCDM) [22, 23].

The paper is structured as following: paragraph 2 covers statement of the problem; paragraph 3 introduces the algorithm for solving the problem; in the last paragraph, some extracts from computation of SCI are provided.

2. Statement of the problem

In this paper, the main idea is to present a methodology based on interval-valued intuitionistic fuzzy instruments for the computation purpose of SCI. With this intention, a MCDM algorithm is developed based on IVIFS. When selecting indicators for calculating the SCI, the methodology of UN in Latin America is used [24]. An example of SCI calculation is based on Azerbaijan data for 2021, collected from some reliable sources [25-27], which is provided in Table 2.

In order to fuzzify the SCI indicators, the best and worst cases are selected from the 2021 data for countries worldwide. This allows identification of the lower and upper bounds of each universe of discourse for SCI indicators as fuzzy variables, which are also given in Table 2.

Table 2

SCI DATA ON AZERBAIJAN

№	Social cohesion sub-indices and indicators	2021		
		Actual data	**Worst case**	**Best case**
1	**Distances**			
1.1	Undernourishment (UND)	2.50	55.70	2.50
1.2	Unemployment (UNE)	6.00	29.81	0.09
1.3	Public services index (PSI)	5.50	10.00	0
1.4	Secondary school attainment (SSA)	95.60	0	100.00
1.5	Life expectancy (LEX)	67	53	85
1.6	High tech exports, percent of manufactured exports (HTE)	2.10	0	70.54
2	**Inclusion-exclusion mechanisms**			
2.1	Civil liberties and political rights (CLP)	10	0	100
2.2	Perception of corruption (POC)	30	100	0
2.3	Number of taxes (NOT)	7	99	3
2.4	Public spending on education, percent of public spending (PSE)	15.80	4.36	33.83
3	**Sense of belonging**			
3.1	Percentage of women in parliament (PWP)	18.18	0	61.25
3.2	Social capital index (SC)	47.20	29.90	66.00
3.3	Government effectiveness index (GEI)	0.25	-2.50	2.50
3.4	Voice and accountability index (VAI)	-1.53	-2.50	2.50
3.5	Death from interpersonal violence (death/100000) (DIV)	3.39	103.60	0

3. An algorithm for computation of SCI

The algorithm developed for evaluation of SCI is introduced below.

***Step 1*. Interval-valued intuitionistic fuzzification of crisp input data.** For the fuzzification of the data, interval-valued intuitionistic fuzzification triangular membership function is employed. To enhance clarity, the following definitions on intuitionistic fuzzy sets and its extension – IVIFS are given below:

Definition 1 (Atanassov [20]). Let X represent an universal set. An intuitionistic fuzzy set $\tilde{A}$ in X can be expressed as $\tilde{A} = \{(x, \mu_A(x), v_A(x)): x \in X\}$, where $\mu_A(x)$: $X \to [0, 1]$ and $v_A(x)$: $X \to [0, 1]$ determine the extent of membership and non-membership, respectively, for each element $x \in X$. It holds true for every $x \in X$ that $0 \leq \mu_A(x) + v_A(x) \leq 1$. The value of $\pi_A(x) = 1 - \mu_A(x) - v_A(x)$ is termed as the degree of non-determinacy (or uncertainty) of the element $x \in X$ to the intuitionistic fuzzy set $\tilde{A}$. If $\pi_A(x) = 0$, the intuitionistic fuzzy set reduces to a fuzzy set, taking the form $\tilde{A} = \{(x, \mu_A(x), 1 - \mu_A(x))\}$.

Definition 2 (Atanassov and Gargov [21]). An interval-valued intuitionistic fuzzy set $\tilde{A}$ in X is defined as $\tilde{A} = \{(x, [\mu_A^-(x), \mu_A^+(x)], [v_A^-(x), v_A^+(x)]): x \in X\}$, where $\mu_A^-(x)$: $X \to [0,1], \mu_A^+(x): X \to [0,1]$ specify the lower and upper degrees of membership, and $v_A^-(x): X \to [0,1]$, $v_A^+(x): X \to [0,1]$ denote the lower and upper degrees of non-membership for the element $x \in X$. Moreover, for every $x \in X$, it holds that $0 \leq \mu_A^+(x) + v_A^+(x) \leq 1$. The lower and upper degrees of non-determinacy are defined as $\pi_A^-(x) = 1 - \mu_A^+(x) - v_A^+(x)$ and $\pi_A^+(x) = 1 - \mu_A^-(x) - v_A^-(x)$, respectively.

Definition 3 (Bharati [28] with our clarifications). An interval-valued intuitionistic fuzzy number is represented as: $\tilde{A} = \{(a, b, c): [\mu_A^-, \mu_A^+], [v_A^-, v_A^+]\}$, where $\mu_A^-: X \to [0,1]$, $\mu_A^+: X \to [0,1]$ denote the lower and upper membership degrees in the top of $\tilde{A}$ (where $x = b$), and $v_A^-: X \to [0,1]$, $v_A^+: X \to [0,1]$ denote the lower and upper non-membership degrees in $x = b$. Thus, for each element $x \in X$ the interval-valued intuitionistic fuzzification triangular memberships and non-memberships can be calculated as:

$$\mu_A^-(x) = \begin{cases} \mu_A^- \dfrac{(x-a)}{(b-a)}, & a \leq x < b, \\ \mu_A^-, & x = b, \\ \mu_A^- \dfrac{(c-x)}{(c-b)}, & b < x \leq c, \end{cases} \tag{1}$$

$$\mu_A^+(x) = \begin{cases} \mu_A^+ \dfrac{(x-a)}{(b-a)}, & a \le x < b, \\ \mu_A^+, & x = b, \\ \mu_A^+ \dfrac{(c-x)}{(c-b)}, & b < x \le c, \end{cases} \tag{2}$$

$$v_A^-(x) = \begin{cases} 1-(1-v_A^-)\dfrac{(x-a)}{(b-a)}, & a \le x < b, \\ v_A^-, & x = b, \\ v_A^- + (1-v_A^-)\dfrac{(x-b)}{(c-b)}, & b < x \le c, \end{cases} \tag{3}$$

$$v_A^+(x) = \begin{cases} 1-(1-v_A^+)\dfrac{(x-a)}{(b-a)}, & a \le x < b, \\ v_A^+, & x = b, \\ v_A^+ + (1-v_A^+)\dfrac{(x-b)}{(c-b)}, & b < x \le c. \end{cases} \tag{4}$$

For the evaluation of the SCI, we propose a novel approach to the fuzzification of SCI indicators, within the framework of which the triangular membership functions (1), (2), and non-membership functions (3) and (4) will be right-angled. In this case, for positively influencing factors (when their growth will also increase the function value), the triangular membership function will have a rising hypotenuse (in fact, only the left side of the right triangle is taken, so $b = c$). And for negatively influencing factors, only the right side of the triangle is taken, when $b = a$.

***Step 2*. Forming the interval-valued intuitionistic fuzzy preference relation matrix (IVIFPRM).**

In this step, interval-valued intuitionistic fuzzy numbers (IVIFNs) are formed, which implement a set of criteria preferences described by linguistic terms with corresponding acronyms (see Table 3), based on the scale proposed in [29] as an IVIFS extension of comparison scale of criteria for the fuzzy analytic hierarchy process.

Table 3

INTERVAL-VALUED INTUITIONISTIC FUZZY NUMBERS FOR CRITERIA PREFERENCES

Linguistic term	Acronym	IVIFN
Absolutely Important	AI	([0.81, 0.90],[0.00,0.10])
Highly Important	HI	([0.71, 0.80],[0.11, 0.20])
Important	I	([0.61, 0.70],[0.21, 0.30])
Medium Important	MI	([0.51, 0.60],[0.31, 0.49])
Equally Important	EI	([0.50, 0.50],[0.50, 0.50])
Medium Unimportant	MU	([0.31, 0.49],[0.51, 0.60])
Unimportant	U	([0.21, 0.30],[0.61, 0.70])
Highly Unimportant	HU	([0.11, 0.20],[0.71, 0.80])
Absolutely Unimportant	AU	([0.00, 0.10],[0.81, 0.90])

Using the corresponding interval-valued intuitionistic fuzzy numbers matching the linguistic terms provided in Table 3, the IVIFPRM is constructed for each sub-index of SCI by analogy with the Saaty pairwise comparison matrix:

$$R = \begin{matrix} & x_1 & x_2 & \cdots & x_n \\ x_1 & r_{11} & r_{12} & \cdots & r_{1n} \\ x_2 & r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ x_n & r_{n1} & r_{n2} & \cdots & r_{nn} \end{matrix},$$

where $r_{ij} = [\underline{r}_{ij}, \bar{r}_{ij}]$ stands for the preference degree intervals of sub-indices or indicators x_i over x_j, n is the number of indicators included in the corresponding sub-index or the number of sub-indices for calculating the overall index.

***Step 3*. Checking the additive consistency.** In IVIFPRM, getting valid computation results depends on matrix consistency. Since the weak consistency may lead to distorted results, it is approved as a key problem in IVIFPRMs. Then, additive conditions [30] given below for $R = (r_{ij})_{n\times n} = \left([\mu_{ij}^L, \mu_{ij}^U], [v_{ij}^L, v_{ij}^U]\right)_{n\times n}$ must hold:

$$\left[\mu_{ij}^L, \mu_{ij}^U\right] \subseteq [0,1], \left[v_{ij}^L, v_{ij}^U\right] \subseteq [0,1], 0 \le \mu_{ij}^U + v_{ij}^U \le 1,$$
$$\left[\mu_{ij}^L, \mu_{ij}^U\right] = \left[v_{ij}^L, v_{ij}^U\right] = [0.5, 0.5]\ \forall\, i, j = 1,2,\ldots,n, \quad (5)$$

where μ_{ij}^L, and μ_{ij}^U specify the lower and upper degrees of membership, and v_{ij}^L, and v_{ij}^U denote the lower and upper degrees of non-membership for the elements r_{ij} of IVIFPRM.

***Step 4*. Constructing the multiplicative consistent IVIFPRM.** It has been proven that the additive consistency of IVIFPRM is not sufficient, so it is necessary to ensure the multiplicative consistency by constructing the corresponding interval-valued intuitionistic fuzzy preference relation matrix $\bar{R} = \left([\bar{\mu}_{ij}^L, \bar{\mu}_{ij}^U], [\bar{v}_{ij}^L, \bar{v}_{ij}^U]\right)_{n\times n}$ applying the following equations [31] subject to $j > i + 1$:

$$\bar{\mu}_{ij}^L = \frac{\sqrt[j-i-1]{\prod_{k=i+1}^{j-1} \mu_{ik}^L \mu_{kj}^L}}{\sqrt[j-i-1]{\prod_{k=i+1}^{j-1} \mu_{ik}^L \mu_{kj}^L} + \sqrt[j-i-1]{\prod_{k=i+1}^{j-1} (1-\mu_{ik}^L)(1-\mu_{kj}^L)}}, \quad (6)$$

$$\bar{\mu}_{ij}^U = \frac{\sqrt[j-i-1]{\prod_{k=i+1}^{j-1} \mu_{ik}^U \mu_{kj}^U}}{\sqrt[j-i-1]{\prod_{k=i+1}^{j-1} \mu_{ik}^U \mu_{kj}^U} + \sqrt[j-i-1]{\prod_{k=i+1}^{j-1} (1-\mu_{ik}^U)(1-\mu_{kj}^U)}}, \quad (7)$$

$$\bar{v}_{ij}^L = \frac{\sqrt[j-i-1]{\prod_{k=i+1}^{j-1} v_{ik}^L v_{kj}^L}}{\sqrt[j-i-1]{\prod_{k=i+1}^{j-1} v_{ik}^L v_{kj}^L} + \sqrt[j-i-1]{\prod_{k=i+1}^{j-1} (1-v_{ik}^L)(1-v_{kj}^L)}}, \quad (8)$$

$$\bar{v}_{ij}^U = \frac{\sqrt[j-i-1]{\prod_{k=i+1}^{j-1} v_{ik}^U v_{kj}^U}}{\sqrt[j-i-1]{\prod_{k=i+1}^{j-1} v_{ik}^U v_{kj}^U} + \sqrt[j-i-1]{\prod_{k=i+1}^{j-1} (1-v_{ik}^U)(1-v_{kj}^U)}}, \quad (9)$$

where $\bar{\mu}_{ij}^{L}: X \to [0,1]$, $\bar{\mu}_{ij}^{U}: X \to [0,1]$ denote the lower and upper membership degrees, and $\bar{v}_{ij}^{L}: X \to [0,1]$, $\bar{v}_{ij}^{U}: X \to [0,1]$ denote the lower and upper non-membership degrees of the elements in multiplicative consistent interval-valued intuitionistic fuzzy preference relation matrix $\bar{R}$.

***Step 5*. Computation of Entropy.** Next, we calculate the entropy e_{ij} of the elements in multiplicative consistent interval-valued intuitionistic fuzzy preference relation matrix $\bar{R}$ applying the method proposed by Yager [32], with the following equation:

$$e_{ij} = \frac{2 - \left|\bar{\mu}_{ij}^{L} + \bar{\mu}_{ij}^{U} - \bar{v}_{ij}^{L} - \bar{v}_{ij}^{U}\right| + \bar{\pi}_{ij}^{L} + \bar{\pi}_{ij}^{U}}{2 + \left|\bar{\mu}_{ij}^{L} + \bar{\mu}_{ij}^{U} - \bar{v}_{ij}^{L} - \bar{v}_{ij}^{U}\right| + \bar{\pi}_{ij}^{L} + \bar{\pi}_{ij}^{U}}, \tag{10}$$

where $\bar{\pi}_{ij}^{L}: X \to [0,1]$, $\bar{\pi}_{ij}^{U}: X \to [0,1]$ denote the lower and upper degrees of non-determinacy of the elements in multiplicative consistent interval-valued intuitionistic fuzzy preference relation matrix $\bar{R}$.

***Step 6*. Construction of Entropy matrix.** Based on calculations by formula (10) applied to the multiplicative consistent IVIFPRM, we obtain the entropy matrix:

$$E = \begin{array}{c} \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{pmatrix} e_{11} & e_{12} & \cdots & e_{1n} \\ e_{21} & e_{22} & \cdots & e_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ e_{n1} & e_{n2} & \cdots & e_{nn} \end{pmatrix} \end{array}.$$

***Step 7*. Obtaining the criteria weights.** Firstly, the entropy information measures of each criterion selected for assessing the SCI, are computed with the following equations [33]:

$$E_j = \frac{1}{n}\sum_{i=1}^{n} e_{ij}, \quad j = \overline{1,n}. \tag{11}$$

Next, the criteria weights are computed using the following equation:

$$w_j = \frac{1 - E_j}{\sum_{j=1}^{n}(1 - E_j)}, \quad j = \overline{1, n}. \tag{12}$$

***Step 8*. Aggregation of indicators and sub-indices.** In this step, interval-valued intuitionistic fuzzy weighted aggregation operator (IIFWA) [34] is employed to combine interval-valued intuitionistic fuzzy numbers of SCI indicators to calculate the sub-indices (13) and the overall value of SCI as IVIFNs (14) based on them.

$$IIFWA_s = \left(\left[1 - \prod_{j=1}^{n}\left(1 - \mu_j^-\right)^{w_j}, 1 - \prod_{j=1}^{n}\left(1 - \mu_j^+\right)^{w_j}\right], \left[\prod_{j=1}^{n}\left(v_j^-\right)^{w_j}, \prod_{j=1}^{n}\left(v_j^+\right)^{w_j}\right]\right), \quad \overline{1, k}, \tag{13}$$

where k is the number of sub-indices (3 in our case – Distances, Inclusion-exclusion mechanisms, and Sense of belonging), n is the number of indicators for the sub-index being computed (changing for each sub-index), $w_1, \dots, w_n$ are weights of indicators of corresponding sub-index, $([\mu_j^-, \mu_j^+], [v_j^-, v_j^+])$ is IVIFN for indicator j.

As a result, we get aggregated IVIFN for each sub-index as $IIFWA_s = ([\mu_s^-, \mu_s^+], [v_s^-, v_s^+])$. Based on them, we obtain overall SCI assessment as interval-valued intuitionistic fuzzy weighted aggregation:

$$IIFWA = \left(\left[1 - \prod_{s=1}^{k}(1 - \mu_s^-)^{w_s}, 1 - \prod_{s=1}^{k}(1 - \mu_s^+)^{w_s}\right], \left[\prod_{s=1}^{k}(v_s^-)^{w_s}, \prod_{s=1}^{k}(v_s^+)^{w_s}\right]\right), \quad \overline{1, k}, \tag{14}$$

where $w_1, \dots, w_s$ are weights of sub-indices (w_1 for Distances, w_2 for Inclusion-exclusion mechanisms, and w_3 for Sense of belonging).

As a result of applying (14), we obtained an interval-valued intuitionistic fuzzy weighted aggregation operator for SCI $IIFWA = ([\mu_{IIFWA}^{-}, \mu_{IIFWA}^{+}], [v_{IIFWA}^{-}, v_{IIFWA}^{+}])$.

***Step 9*. Establishment of interval-valued intuitionistic fuzzy scale.** With the purpose to recognize the level of aggregated interval-valued intuitionistic fuzzy numbers among the set of linguistic terms, we developed the scale according to [35] that is given in Table 4.

Table 4

LINGUISTIC TERMS AND THEIR MATCHING INTERVAL-VALUED INTUITIONISTIC FUZZY SCALE

Linguistic term	Acronym	IVIFN
Very High	VH	([0.86, 0.95], [0.00, 0.05])
High	H	([0.71, 0.85], [0.06, 0.15])
Medium High	MH	([0.56, 0.70], [0.16, 0.30])
Medium	M	([0.45, 0.55], [0.31, 0.45])
Medium Low	ML	([0.30, 0.44], [0.46, 0.56])
Low	L	([0.15, 0.29], [0.57, 0.71])
Very Low	VL	([0.00, 0.14], [0.72, 0.86])

***Step 10*. Calculation of similarity measures.** In the final step, in order to recognize the pattern of aggregated interval-valued intuitionistic fuzzy numbers of SCI among the set of linguistic terms given in Table 4, similarity measures are calculated.

Since *IIFWA* is an IVIFN, as all fuzzy sets in Table 4 are presented in a similar form, then the Wei measure $S_W(IIFWA, C)$ can be used to determine the similarity between aggregated SCI value (*IIFWA*) and IVIFNs corresponding to each linguistic term $C \in$ {VH, H, MH, M, ML, L, VL} given in Table 4, using the following equation [36]:

$$S_W(IIFWA, C) = \\ = \frac{1}{k}\sum_{i=1}^{k}\frac{2-\min\left(\mu_i^{-}(x_i), v_i^{-}(x_i)\right)-\min\left(\mu_i^{+}(x_i), v_i^{+}(x_i)\right)}{2-\max\left(\mu_i^{-}(x_i), v_i^{-}(x_i)\right)-\max\left(\mu_i^{+}(x_i), v_i^{+}(x_i)\right)}, \tag{15}$$

where k is the number of pairs between interval-valued intuitionistic fuzzy numbers *IIFWA* and *C* (which is the IVIFN of each of linguistic terms from Table 4), the values of interval-valued intuitionistic fuzzification triangular memberships and non-memberships at each point *i* are calculated in accordance with formulas (1)-(4), and:

$$\mu_i^-(x_i) = |\mu_{IIFWA}^-(x_i) - \mu_C^-(x_i)|,$$

$$\mu_i^+(x_i) = |\mu_{IIFWA}^+(x_i) - \mu_C^+(x_i)|,$$

$$v_i^-(x_i) = |v_{IIFWA}^-(x_i) - v_C^-(x_i)|,$$

$$v_i^+(x_i) = |v_{IIFWA}^+(x_i) - v_C^+(x_i)|.$$

4. SCI computation results

In Table 5 we present the actual data of the analyzed country – Azerbaijan in 2021, converted into interval-valued intuitionistic fuzzy numbers. Below, as an example, we apply interval-valued intuitionistic fuzzification for the positive Percentage of women in parliament indicator. For this purpose, equations (1)-(4) were applied taking into account the method proposed in step 1. In this case, a right triangle with an ascending hypotenuse is used for the positively influencing indicator (then *b* = *c*). Fuzzy boundaries are taken from worst and best cases in Table 2.

$$\mu_{PWP}^-(18.18) = 0.90 * \left(\frac{18.18 - 0}{61.25 - 0}\right) = 0.2671,$$

$$\mu_{PWP}^+(18.18) = 0.95 * \left(\frac{18.18 - 0}{61.25 - 0}\right) = 0.2820,$$

$$v_{PWP}^-(18.18) = 1 - (1 - 0.01) * \left(\frac{18.18 - 0}{61.25 - 0}\right) = 0.7062,$$

$$v_{PWP}^+(18.18) = 1 - (1 - 0.25) * \left(\frac{18.18 - 0}{61.25 - 0}\right) = 0.7106.$$

These calculations are based on equations (1)-(4), where $\mu^- = 0.90$, $\mu^+ = 0.95$, $v^- = 0.01$, $v^+ = 0.25$ are the lower and upper membership and non-membership degrees, representing the lower and upper approximations provided by experts [37]. These parameters express the accuracy level of statistical information, as outlined in [38]. The result of these calculations can be seen in item 3.1 of Table 5.

Table 5

INPUT DATA CONVERTED INTO INTERVAL-VALUED INTUITIONISTIC FUZZY NUMBERS

№	Social cohesion sub-indices and indicators	2021	
		Actual data	**IVIFN**
1	**Distances**		
1.1	Undernourishment	2.50	([0.90, 0.95], [0.01, 0.03])
1.2	Unemployment	6.00	([0.72, 0.76], [0.21, 0.22])
1.3	Public services index	5.50	([0.41, 0.43], [0.55, 0.56])
1.4	Secondary school attainment	95.60	([0.86, 0.91], [0.05, 0.07])
1.5	Life expectancy	67	([0.39, 0.42], [0.57, 0.57])
1.6	High tech exports, percent of manufactured exports	2.10	([0.03, 0.03], [0.97, 0.97])
2	**Inclusion-exclusion mechanisms**		
2.1	Civil liberties and political rights	10	([0.09, 0.10], [0.90, 0.90])
2.2	Perception of corruption	30	([0.63, 0.67], [0.31, 0.32])
2.3	Number of taxes	7	([0.86, 0.91], [0.05, 0.07])
2.4	Public spending on education, percent of public spending	15.80	([0.35, 0.37], [0.62, 0.61])
3	**Sense of belonging**		
3.1	Percentage of women in parliament	18.18	([0.27, 0.28], [0.71, 0.71])
3.2	Social capital index	47.20	([0.43, 0.46], [0.52, 0.53])
3.3	Government effectiveness index	0.25	([0.50, 0.52], [0.45, 0.46])
3.4	Voice and accountability index	-1.53	([0.17, 0.18], [0.80, 0.81])
3.5	Death from interpersonal violence (death/100000)	3.39	([0.87, 0.92], [0.04, 0.06])

This section provides an example of calculations for Sense of belonging sub-index. Following the conversion of crisp data into interval-valued intuitionistic fuzzy values, IVIFPRM and consistent IVIFPRM are constructed as below according to steps 2–4:

$$R = \begin{array}{c} \\ PWP \\ SC \\ GEI \\ VAI \\ DIV \end{array} \begin{pmatrix} PWP & SC & GEI & VAI & DIV \\ ([0.45,0.55],[0.45,0.55]) & ([0.56,0.70],[0.30,0.44]) & ([0.56,0.70],[0.30,0.44]) & ([0.71,0.85],[0.15,0.29]) & ([0.86,1.00],[0.00,0.14]) \\ ([0.30,0.44],[0.56,0.70]) & ([0.45,0.55],[0.45,0.55]) & ([0.45,0.55],[0.45,0.55]) & ([0.56,0.70],[0.30,0.44]) & ([0.71,0.85],[0.15,0.29]) \\ ([0.30,0.44],[0.56,0.70]) & ([0.45,0.55],[0.45,0.55]) & ([0.45,0.55],[0.45,0.55]) & ([0.56,0.70],[0.30,0.44]) & ([0.56,0.70],[0.30,0.44]) \\ ([0.15,0.29],[0.71,0.85]) & ([0.30,0.44],[0.56,0.70]) & ([0.30,0.44],[0.56,0.70]) & ([0.45,0.55],[0.45,0.55]) & ([0.56,0.70],[0.30,0.44]) \\ ([0.00,0.14],[0.86,1.00]) & ([0.15,0.29],[0.71,0.85]) & ([0.30,0.44],[0.56,0.70]) & ([0.30,0.44],[0.56,0.70]) & ([0.45,0.55],[0.45,0.55]) \end{pmatrix},$$

$$\bar{R} = \begin{array}{c} \\ PWP \\ SC \\ GEI \\ VAI \\ DIV \end{array} \begin{pmatrix} PWP & SC & GEI & VAI & DIV \\ ([0.45,0.55],[0.45,0.55]) & ([0.56,0.70],[0.30,0.44]) & ([0.51,0.74],[0.26,0.49]) & ([0.62,0.84],[0.16,0.38]) & ([0.73,0,90],[0.10,0.27]) \\ ([0.30,0.44],[0.56,0.70]) & ([0.45,0.55],[0.45,0.55]) & ([0.45,0.55],[0.45,0.55]) & ([0.51,0.74],[0.26,0.49]) & ([0.56,0.80],[0.20,0.44]) \\ ([0.26,0.49],[0.51,0.74]) & ([0.45,0.55],[0.45,0.55]) & ([0.45,0.55],[0.45,0.55]) & ([0.56,0.70],[0.30,0.44]) & ([0.62,0.84],[0.16,0.38]) \\ ([0.16,0.38],[0.62,0.84]) & ([0.26,0.49],[0.51,0.74]) & ([0.30,0.44],[0.56,0.70]) & ([0.45,0.55],[0.45,0.55]) & ([0.56,0.70],[0.30,0.44]) \\ ([0.10,0.27],[0.73,0,90]) & ([0.20,0.44],[0.56,0.80]) & ([0.16,0.38],[0.62,0.84]) & ([0.30,0.44],[0.56,0.70]) & ([0.45,0.55],[0.45,0.55]) \end{pmatrix}.$$

In the next step, observing the steps 5 and 6, elements of entropy matrix are assessed:

$$E = \begin{matrix} & \begin{matrix} PWP & SC & GEI & VAI & DIV \end{matrix} \\ \begin{matrix} PWP \\ SC \\ GEI \\ VAI \\ DIV \end{matrix} & \begin{pmatrix} 1.00 & 0.69 & 0.52 & 0.36 & 0.24 \\ 0.69 & 1.00 & 0.69 & 0.52 & 0.36 \\ 0.52 & 0.69 & 1.00 & 0.69 & 0.52 \\ 0.36 & 0.52 & 0.69 & 1.00 & 0.69 \\ 0.24 & 0.36 & 0.52 & 0.69 & 1.00 \end{pmatrix} \end{matrix}.$$

Following the construction of entropy matrix, the weights of all indicators of the Sense of belonging sub-index are calculated according to equations given in step 7:

$$E_1 = 0.5618,\ E_2 = 0.6518,\ E_3 = 0.6839,\ E_4 = 0.6518,\ E_5 = 0.5618,$$

$$w_1 = 0.2320,\ w_2 = 0.1844,\ w_3 = 0.1674,\ w_4 = 0.1844,\ w_5 = 0.2320.$$

Next, the interval-valued intuitionistic fuzzy weighted aggregation operator is computed for all sub-indices. Here is an example calculation for Sense of belonging based on (13):

$$\begin{aligned} IIFWA_3 = {} & ([1 - ((1-0.27)^{0.2320} * (1-0.43)^{0.1844} * \\ & * (1-0.50)^{0.1674} * (1-0.17)^{0.1844} * \\ & * (1-0.87)^{0.2320}), (1-(1-0.28)^{0.2320} * \\ & * (1-0.46)^{0.1844} * (1-0.52)^{0.1674} * \\ & * (1-0.18)^{0.1844} * (1-0.92)^{0.2320})], [(0.71^{0.2320} * \\ & * 0.52^{0.1844} * 0.45^{0.1674} * 0.80^{0.1844} * \\ & * 0.04^{0.2320}), (0.71^{0.2320} * 0.53^{0.1844} * 0.46^{0.1674} * \\ & * 0.81^{0.1844} * 0.06^{0.2320})] = \\ & = ([0.55, 0.61], [0.33, 0.36]). \end{aligned}$$

Following the algorithm, after calculating all sub-indices, the SCI is aggregated based on them in the same way:

$$IIFWA = ([0.61, 0.66], [0.26, 0.29]).$$

Finally, similarity measures between aggregated value for SCI and IVIFN of each linguistic term given in Table 4 are computed based on (15), which can be seen in Table 6.

Table 6

COMPUTED SIMILARITY VALUES OF SCI WITH LINGUISTIC TERMS

Linguistic term	Similarity value
Very high (VH)	0.5922
High (H)	0.7364
Medium high (MH)	0.9065
Medium (M)	0.7931
Medium low (ML)	0.6124
Low (L)	0.4583
Very low (VL)	0.3208

Obviously, the highest similarity value corresponds to the linguistic term Medium high (MH), which allows us to identify the level of SCI of the Republic of Azerbaijan in 2021 as medium high. At the same time, we get an assessment of the sub-indices and indicators on the basis of which it is calculated, which allows us to implement informed policies towards improving the Social cohesion index.

Conclusions

The Social Cohesion Index is one of the sub-indices (social security, social empowerment, social inclusion, and social cohesion) of the Social Quality Index. For the assessment of social sustainability, computing social cohesion is a pressing contemporary problem. The study presents an innovative approach to assessing social cohesion using an interval-valued intuitionistic fuzzy pattern recognition model. This method addresses the inherent uncertainties in social cohesion indicators by applying fuzzy logic, specifically through interval-valued intuitionistic fuzzy numbers. The methodology offers a significant improvement over traditional approaches by better handling the imprecision in data and enhancing the robustness of the Social Cohesion Index calculation.

By applying this interval-valued intuitionistic fuzzy pattern recognition model, we were able to quantify social cohesion in

Azerbaijan using 2021 data. The results demonstrate the effectiveness of this approach in capturing the nuances of social cohesion, particularly in the context of accounting for complex socio-economic variables. The study also established a framework for calculating the weights of various indicators and aggregating them into a composite Social Cohesion Index, which was then assessed using a fuzzy linguistic scale.

The results of investigation for Azerbaijan's performance in social cohesion for 2021 have led to a medium high rating, which indicates that the country performs better than many countries with low social cohesion but is still behind from leading countries. This assessment provides a comprehensive understanding of Azerbaijan's social cohesion relative to global standards, highlighting areas for policy focus and further research. For example, the fuzzy model revealed that there is potential for improvement in governance, inclusion-exclusion mechanisms, social representation, reducing inequality and increasing social capital.

Overall, this research contributes to the broader field of social processes assessment by providing a more nuanced and flexible tool. It highlights the importance of incorporating fuzzy logic into social science methodology, especially when dealing with complex and uncertain datasets. The proposed model not only advances the analytical approach to Social Cohesion Index calculations, but also offers practical insights for policymakers aiming to evaluate and enhance social cohesion within their societies.

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